Chapter 11

Functional Dependencies

Introduction

- Functional dependency (FD) is a relationship from one set of attributes to another
- For examples:
  - In Supplier-Part-Project (SPJ), the set of attributes {S#, P#, J#} → the set of attribute {QTY}
  - For any given values for the triple of attributes {S#, P#, J#}, there is just one corresponding value of attribute {QTY}, but
  - Many distinct values of {S#, P#, J#} can have the same corresponding value for attribute QTY

Basic Definitions

SCP

S#	CITY	P#	QTY
S1	London	P1	100
S1	London	P2	100
S2	Paris	P1	200
S2	Paris	P2	200
S3	Paris	P2	300
S4	London	P2	400
S4	London	P4	400
S4	London	P5	400

Definition of FD

- Let r be a relation, and let X and Y be arbitrary subsets of the set of attributes of r. Then we say that Y is functionally dependent on X

  $X \rightarrow Y$

  ("X functionally determines Y") if and only if each X value in r has associated with it precisely on one Y value in r. In other words, whenever two tuples of r agree on their X value, they also agree on their Y value.

  {S#} → {CITY}

  Because every tuple of that relation with a given S# value also has the same CITY value.

Example of FDs

SCP TABLE:

{S#, P#} → QTY

{S#, P#} → CITY

{S#, P#} → {CITY, QTY}

{S#, P#} → S#

{S#, P#} → {S#, P#, CITY, QTY}

{S#} → {CITY}

- If X is a candidate key of relvar R, the all attributes Y of relvar R must be functionally dependent on X

  {P#} → {P#, PNAME, COLOR, WEIGHT, CITY}

Trivial and Nontrivial Dependencies

- A FD is trivial if an only if the right-hand side is a subset of the left-hand side

  {S#, P#} → {S#}

- Nontrivial
- We are only interested in nontrivial FDs

Closure of A Set of Dependencies

- Some FDs might imply others

  {S#, P#} → {CITY, QTY}

  Implies

  {S#, P#} → {CITY}

  {S#, P#} → {QTY}

- Closure:
  - The set of all FDs that implied by a given set S of FDs is called the closure of S
  - Armstrong's Axioms
    - Then new FDs can be inferred from given ones

Inference Rules

1. Reflexivity: If B is a subset of A, then A →B
2. Augmentation: If A → B, then AC → BC
3. Transitivity: If A → B and B → C, then A → C
4. Self-determination: A → A
5. Decomposition: If A → BC, then A → B and A → C
   (Prove)
6. Union: If A → B and A → C, then A → BC
   (Prove)
7. Composition: If A → B and C → D, then AC → BD

Example:

{A} → {B,C}
{B} → {E}
{C,D} → {E, F}

Closure of S:

1. {A} → {B, C} (given)
2. {A} → {C} (decomposition)
3. {A, D} → {C, D} (augmentation)
4. {C, D} → {E, F} (given)
5. {A, D} → {E, F} (3 and 4, transitivity)
6. {A, D} → {F} (5, decomposition)

Irreducible Sets of Dependencies

A set S of FDs is irreducible if an only if it satifies the following three properties:

- The right-hand side of every FD in S involves just one attribute (singleton set)
- The left-hand side (determinant) of every FD in S is irreducible – meaning that no attribute can be discarded from the determinant without changing the closure S(+). It is called left-irreducible
- No FD in S can be discarded from S without changing the closure S (+).

Example

Irreducible

{P#} → {PNAME}
{P#} → {COLOR}
{P#} → {WEIGHT}
{P#} → {CITY}

Not irreducible

{P#, PNAME} → {COLOR}
{P#} → {PNAME}
{P#} → {WEIGHT}
{P#} → {CITY}

Not irreducible

{P#} → {PNAME, COLOR}
{P#} → {WEIGHT}
{P#} → {CITY}

Not irreducible

{P#} → {P#}
{P#} → {PNAME}
{P#} → {COLOR}
{P#} → {WEIGHT}
{P#} → {CITY}

Example

$A \rightarrow BC$
$B \rightarrow C$
$A \rightarrow B$
$AB \rightarrow C$
$AC \rightarrow D$

$A \rightarrow B$
$A \rightarrow C$
$B \rightarrow C$
$A \rightarrow B$
$AB \rightarrow C$
$AC \rightarrow D$

Irreducible set

$A \rightarrow B$
$B \rightarrow C$
$A \rightarrow D$